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THE

PLANETARY DISTANCES

BY

LAURENCE M'CURRICK

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THE

PLANETARY DISTANCES.

BY

LAURENCE M'CURRICK.

(AUTHOR OF PAPERS ON ELEMENTARY EDUCATION, ETC.)

> "Knowest thou the ordinances of heaven?
> Canst thou set the dominion thereof in the earth?"
> —JOB xxxviii., 33.

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PREFACE.

THE theories propounded in the following pages are in every sense *revolutionary,* and therefore cannot be expected to gain the ascendancy without a struggle. Respecting the superincumbent theory of the Planetary Distances, the uniqueness of the system here unfolded, the variety of methods bringing out the same results, and the great number of remarkable coincidences which cannot reasonably be regarded as accidental, are strong presumptions in its favour.

New light is thrown on the relationship of the circle and square, the sphere and cube; and the progression 1, 3, 9, 27, is found to be their great common highway; a system of Celestial Arithmetic is sketched, showing and comparing the circulates and ratios of circle, square, and number; Bode's Law is shown to be founded on fact, and by its aid the complement or counterpart of Kepler's Third Law is found; the force of factors, both geometrical and numerical, is proved to be the same law of nature as the gravitation of matter; the inverse ratio of the square of the distance is accounted for, and a more benign theory of Solar Influence is put forth.

While the cry is still for more light, this essay may in some measure tend to promote the spread of truth and the increase of knowledge.

THE PLANETARY DISTANCES.

CHAPTER I.

THE GEOMETRICAL CONSTRUCTION OF THE SOLAR SYSTEM.

Method I. (Fig. 1.)

ABOUT the square $m n o p$ circumscribe a circle. S is the centre of the circumscribed circle. Draw the diameter $q a$ parallel to $p m$, and cutting $m n$ in r. Produce $q a$ to x, and bisect $r a$ in v. The radius Sa is taken as the distance of Mercury from the Sun. From $a x$ cut off $a b$ equal to Sv, and $a c$ equal to arc of quadrant $n a m$. Sb is the distance of Venus from the Sun, and Sc, that of the Earth. From $c x$ cut off $c d$ equal to arc of quadrant $t v y$. Sd is the distance of Mars. The quadrant is now exhausted. From $d x$ cut off $d e$ equal to semi-circumference $a m p q$. Se is the average distance of the Asteroids. From $e x$ cut off $e f$ equal to circumference $a m q n$. Sf is the distance of Jupiter. The first circle is now exhausted. The progression is by circles, the radius of each being respectively the 1st, 4th, 7th distance, the difference being 3. From Sx cut off Sg equal to the circumference of the second circle, whose radius is Sd. Sg is the distance of Saturn. The second circle is exhausted at once. Proceed now to the third circle, whose radius is Sg. From Sx cut off Sh equal to the diameter. Sh is the distance of Uranus. From Sx cut off Sk equal to the semi-circumference. Sk is the distance of Neptune. From Sx cut off Sl equal to the circumference. Sl is the distance of the undiscovered planet.

Construct a scale so that S c (the Earth) = 1.

			Present Mean Distances.
Mercury, S a	$= \dfrac{1}{\dfrac{1+3\cdot1416}{2}}$	= ·389	·387099
Venus, S b	= ·389 + ·332	= ·721	·723332
Earth, S c	=	1·000	1·000000
Mars, S d	$= 1 + \left(\cdot332 \times \dfrac{3\cdot1416}{2}\right)$	= 1·5215	1·523691
Asteroids, S e	= 1·5215 + (·389 × 3·1416)	= 2·7437	
Jupiter, S f	= 2·7437 + (·389 × 3·1416 × 2)	= 5·1877	5·202798
Saturn, S g	= 1·5215 × 3·1416 × 2	= 9·5598	9·538852
Uranus, S h	= 9·5598 × 2	= 19·1196	19·182639
Neptune, S k	= 9·5598 × 3·1416	= 30·0331	30·036970
Undiscovered, S l	= 9·5598 × 3·1416 × 2	= 60·0662	

Method II. (Fig. 2.)

By the first method the relative distances are not affected by the particular unit of measure; but by the second method the unit of measure must be the first distance (Mercury) divided by 3·9. This number must be supposed to belong to the Problem by this method of solution, and will be accounted for in a subsequent chapter. The Inner Circle in fig. 2 is divided proportionally, the extremes being the radius and the circumference, and the means being such as to make the sum of the first three proportionals as nearly as possible equal to the fourth; thus—

Rad. $(a) : b :: b : \dfrac{b^2}{a}$ Rad. $: b :: \dfrac{b^2}{a}$: Circumference

3·9 : 7·2 :: 7·2 : 13·3. 3·9 : 7·2 :: 13·3 : 24·5.

Proportionals of Inner Circle, 3·9 : 7·2 :: 13·3 : 24·5.

Difference between, 3·3 6·1 11·2.

Proportionals of Middle Circle, $d : e :: f : g$ = 15·2 : 28 :: 51·9 : 95·6.

Proportionals of Outer Circle, Rad. : Diam. :: Semicir. : Circumf. = 95·6 : 191·2 :: 300 : 600. Rad. = 4th Prop. of Middle Circle. Diam. = Sum of Middle Circle.

Progression of Distances.

1. Radius,	3·9	= Mercury	= 3·9	Inner Circle.
2. Rad. + 1st. Dif.	3·9 + 3·3	= Venus	= 7·2	
3. Rad. + 2nd Dif.	3·9 + 6·1	= Earth	= 10·	
4. Rad. + 3rd Dif.	3·9 + 11·2	= Mars	= 15·1	

Or,

$$\left.\begin{array}{l}\text{4. Rad.} \times \text{1st Prop. } 3{\cdot}9 \times 3{\cdot}9 = \text{Mars} = 15{\cdot}21\\ \text{5. Rad.} \times \text{2nd Prop. } 3{\cdot}9 \times 7{\cdot}2 = \text{Asteroids} = 28{\cdot}08\\ \text{6. Rad.} \times \text{3rd Prop. } 3{\cdot}9 \times 13{\cdot}3 = \text{Jupiter} = 51{\cdot}87\\ \text{7. Rad.} \times \text{4th Prop. } 3{\cdot}9 \times 24{\cdot}5 = \text{Saturn} = 95{\cdot}55\end{array}\right\}\text{Middle Circle.}$$

Or,

$$\left.\begin{array}{l}\text{7. Circumf.} \times \text{Rad. } 24{\cdot}5 \times 3{\cdot}9 = \text{Saturn} = 95{\cdot}55\\ \text{8. Circumf.} \times \text{Diam. } 24{\cdot}5 \times 7{\cdot}8 = \text{Uranus} = 191{\cdot}1\\ \text{9. Circumf.} \times \text{Semicir. } 24{\cdot}5 \times 12{\cdot}25 = \text{Neptune} = 300{\cdot}125\\ \text{10. Circumf.} \times \text{Circumf. } 24{\cdot}5 \times 24{\cdot}5 = \text{Undisc.} = 600{\cdot}125\end{array}\right\}\text{Outer Circle.}$$

While the last distance in the beautiful arrangement here disclosed evidently ends the Planetary System, it is by no means so clear that the first begins it. During a total eclipse of the sun on 6th May this year (1883) efforts were to be made to ascertain if there are intra-Mercurial planets. We may here anticipate what will be shown in a subsequent chapter so far as to state that if there are such planets, they are two in number, the second (S *r*, fig. 1) being twice the distance from the Sun that the first is, and Mercury, the third (*c*), bearing the following relation to them:

$$a : b :: b^2 : c^2 ;\ \text{or, } a : 2a :: 4a^2 : c^2.$$

There is thus reason to hope that our theory may be tested by actual observation.

PROGRESSION OF CIRCLES – DIFFERENCE = 3.

Distance.	
1.	1st Radius = 3·9 = 1st Proportional.
4.	2nd Radius × 3·9 = Cube of 1st Prop.
7.	3rd Radius × 3·9 = Cube of 2nd Prop.
10.	4th Radius × 3·9 = Cube of 3rd Prop.

The cube of the 4th Proportional remains to be accounted for. Since, by the theory, the tenth distance is the last, the orbit of the undiscovered planet, multiplied by 3·9, is the cube of the 4th Proportional.

Regarding these circles as the circumference of spheres, the Planetary System, enclosed as in its shell, is thus completed.

CHAPTER II.

PHYSICAL CAUSE OF THE PLANETARY DISTANCES.

LET the three circles, fig. 2, represent the circumference of the Sun in three successive stages of expansion from heat. A force acting on the Inner Wheel is such as to make it turn on its axis till the first proportional lies on the line S x. Suppose a drop to fall here from the rotating molten mass, and to form Mercury. The force, increasing as the wheel goes round, now sends it round on its axis, and along the line at the same time till the second proportional reaches the line. The drop falls forming Venus. After throwing off each planet, the wheel goes back like a rocking-stone and comes forward with accumulated force. It now revolves till b' is on a, and then runs on to the third proportional, the Earth. The wheel repeats the process till c' is on a, and then runs on to the fourth proportional, Mars. The revolution of the Inner Wheel is now completed. The following approximate ratios of the Inner Wheel show the increasing or accumulating force: b', c', d', indicating the distance between the planets. $a : b' :: b' : c'$; and $b' : c :: b'+c' : d'$.

Sum $(a+b'+c'+d') = d = 15{\cdot}2$. The ratios of the distances from the Sun are: $2{\cdot}8 : 3{\cdot}9 :: 7{\cdot}2 : 10$.

The Middle Wheel, the first proportional of which contains the sum of the distances $(a+b'+c'+d' = 15{\cdot}2)$ of the Inner Wheel, now revolves till its second proportional reaches the line, Asteroids. In a similar way the third proportional marks off Jupiter, and the fourth, Saturn. One revolution of the Middle Wheel is now completed; but as the sum of its distances is (or should be) equal to twice its fourth proportional, the Middle Wheel makes a second revolution at once. The drop falls, Uranus. The ratios of the Middle Wheel are:—

$a : b :: d : e$; and $d : e :: f : g$; then $h = (d+e+f+g.)$

The Outer Wheel has for its first proportional the fourth proportional of the Middle Wheel, and for its second proportional the sum of the distances of the Middle Wheel. Hence, the distances being proportionals, $g : h :: k : l$. But g is the radius, h the diameter, and l the circumference; hence k is the semi-circumference. Again, as in proportion, the product of the extremes is equal to the product of the means, the proportionals on fig. 2 being for the Outer Circle 95·6 : 176·4 :: 326 : 600; hence, 191·2 : 176·4 : : 326 : 300. In other words, the radius and the circumference being the extremes, and the diameter being one of the means, the semi-circumference is necessarily the other. In this way it is found that the accumulated force is now such as to send round the Outer Wheel from its first proportional a distance equal to its second proportional with the second proportional of the Middle Wheel added to it (176·4 + 28). The drop falls, Neptune. The Outer Wheel now completes the revolution, the undiscovered planet being twice the distance of Neptune from the Sun.

As the distances on the Outer Wheel are not marked off directly by the proportionals of fig. 2, like those of the Middle and Inner, it is desirable to see as clearly as possible how they are found. There are two ways of dividing into the proportionals, namely, the one in the figure, and the other by making the distance between the first and second proportionals equal to 7·2; the third proportional thus completing the circle, and the fourth proportional being a second revolution of the circle. If the Middle Wheel recoil or rock back so as to lay each entire distance on the line, the work is equal to two revolutions, while by this method (fig. 2) only one revolution of the wheel is really made. The distances lie doubled on the line, and the second revolution of the Middle Wheel is only as it were the tightening of the line. These two revolutions form the first and second proportionals of the Outer Wheel before it is in motion. The position of the third proportional follows as a matter of course, the fourth being the circumference.

The work done by the several wheels in succession may lead to some knowledge of the nature of the force acting upon them. In the following illustration b is the distance from a; c, the distance from b, etc. First, a generates b; then a and b together generate $b+c$; and $b+c$ generate d; and so on, like compound interest, all the latent or conserved force or energy, or whatever it may be called, being used up.

Work of Inner Wheel not transferred to Middle.	a	3·9	
	b	3·3	
	$b+c$	6·1	
			13·3
	$a+b+c+d$	15·2	
	$a+b+c+d+e$	28·	
	$a+b+c+d+e+f$	51·9	
	$a+b+e+d+e+f+g$	95·6	
	Sum of Middle Wheel	191·2	
			286·8
Outer in motion	Sum of foregoing, Inner and Middle,		300·1
	Total sum of Inner, Middle, and Outer,		600·2

It is often observed that small apparent discrepancies strengthen the proof. The eighth distance may here be taken as an instance of the truth of that observation, though the discrepancies will be more particularly accounted for in a subsequent chapter. It is the first distance not marked off directly by the foregoing proportionals, and is not exactly twice the seventh, as it should be by the theory, being about a half per cent., or one in two hundred more than double. Now it is remarkable that such an excess, greater or less, must be shown by any mechanical illustration, be it wheel, roller, groove of spinning-top, or gathering snowball, the radius of the second revolution being longer than that of the first by the thickness of the layer.

If this investigation does not demonstrate the existence of another planet at twice the distance of Neptune, the presumption that it is either there or nowhere is so strong as to warrant a search. If there is a planet there it will likely be found yet. On the other hand, if the recent search near the Sun has been successful, this theory calls for its testimony to the following arrangement, part of which has yet to be explained.

1.= 1·379
2.= 2·758
3.= 3·9
4.= 7·2
5.= 10·
6.= 15·2
7.= 27·58 (Swivel or recurring number).
8.= 52·
9.= 95·6
10.= 191·2
11.= 300·
12.= 600·
13.=3770· (Orbit of 12th).

We learn from the Scriptures that when the present system shall have run its appointed course, it will be renewed in a similar manner. Wound up like a spinning-top, or rolled together like a scroll, the elements melting with fervent heat, the rotating mass shall again be unfurled with orbs purified and refined—new heavens and a new earth. Then again shall the morning stars sing together, and the sons of God shout for joy. "All thy works shall praise Thee, O Lord, and Thy saints shall bless Thee."

CHAPTER III.

BODE'S LAW AND THE PROPERTIES OF NUMBERS.

ON looking at some of the distances they are seen to advance approximately by squares and tens; thus, $a^2=d$; $b^2=f$; $\frac{b^3}{a}=g$; $c-b=\frac{e}{10}$; $d-c=\frac{f}{10}$; and the Inner Circle corresponds to units, the Middle to tens, and the Outer to hundreds. If the Inner begin with units, as in Method II., the ratio is simple, and the relative distances are preserved; but if it begin with tens, the several successive products must be divided by ten; if it begin with hundreds, they must be divided by hundreds, and so on. For example, beginning with the first distance 3·9, the fourth is 15·21; but beginning with 39, it is 1521, which must be divided by ten to preserve the ratio. Again, 390 for the first gives 152,100 for the fourth, which must be divided by a hundred. This ratio may be thus expressed, 10^0, 10^1, 10^2, 10^3, 10^4, etc. Bode's Law, which previous to the discovery of Neptune was the only approximation by ratio for showing the relation of the distances, is founded on an imperfect knowledge of this ratio, combined with the geometrical fact that the square is doubled by making its diagonal the side. Bode's Law runs thus:—

$$\begin{array}{lcl|lcl}
a & = 4 & = 4 & e & = 4 + 3\cdot2^3 & = 28 \\
b & = 4 + 3\cdot2^0 & = 7 & f & = 4 + 3\cdot2^4 & = 52 \\
c & = 4 + 3\cdot2^1 & = 10 & g & = 4 + 3\cdot2^5 & = 100 \\
d & = 4 + 3\cdot2^2 & = 16 & h & = 4 + 3\cdot2^6 & = 196
\end{array}$$

Neptune breaks through it altogether, being only 300 instead of 388 as by it. The fact of the Asteroids having been discovered by means of this law, now proved to be false, shows that the nature of an hypothesis ought not to be very strictly confined; that in some cases it is beneficial to give free scope to the imagination. On looking back it may now be seen how this so-called law runs so long on the rails. The number 10 is a multiple of 2, and the 3 is the progression by circles; the numbers 2, 3, and 4 are small, and the arrangement is favourable for bringing out a desired result. A recent writer on this subject has hit the mark here by showing how numbers may often be so arranged as to bring out approximately what is wanted. It is not uncommon in dealing with statistics, for example, for the opposing parties to use the same illustration in support of their argument, each making it tell in his own favour. Geometry cannot be so dealt with; hence the present success.

If we keep by even tens the distances may be multiplied and divided as here shown; but if we multiply by such a number as 95, that is, to reduce to English miles, the ratio is destroyed. For example, taking the Earth as 10 millions, the distance of Venus is 7,233,320, and that of Jupiter 52,027,980; the square of Venus, divided by the requisite power of 10, as shown above, is 52,320,918, that is, a very little more than the distance of Jupiter. The square root of Jupiter's distance, similarly regulated by this decimal ratio, is 7,213,040, that is, very near the distance of Venus. If we multiply each number by 95 these relations are changed, Venus becoming (7,233,320 × 95), 687,165,400; and Jupiter (52,027,980 × 95) 4,942,658,100. Now, the square of 687,165,400 is 4,721,962,869, which is considerably less than Jupiter's distance. To bring the square of Venus in English miles to correspond to the distance of Jupiter, the number must be raised to above 700 millions instead of 687 millions.

Let us now take the prime factor 3·9, and see what may be made of it. Suppose the 3 to represent the cube, the three sides of which multiplied together give its solidity, the multiplication being as by logarithms $1 + 1 + 1 = 3$; and the ·9 to represent the square divided by 10; further, the side of the cube to correspond to the diameter of the sphere, and the side of the square to that of the circle. In the proportion 1 : 3 : : 9 : 27, the last term inverted and divided by 10 (7·2) may represent the second distance. This may appear rather fanciful; but when the wonderful adaption of the positions and combinations of numbers, revolving on the cipher, like the Earth on its axis, or like the planets round the Sun, as shown by what are called magic squares, the special properties of the "nines," calculations by logarithms, and various other processes, is taken into account, the idea is not unreasonable or absurd. We have only to look at Methods I. and II. to see that the same results are brought out by addition in the one, and by multiplication in the other. Such a method of calculation as is here indicated, showing the relation between the circle and square, the sphere and cube, and the relative positions and distances of the planets, produced by revolution, is here about to be sketched under the name of "Celestial Arithmetic."

Let us look for a little at Multiplication Table. Taking only what applies to our subject, we see that the diagonal line from 1 to 100 shows the squares of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, respectively, and corresponds to the areas of concentric squares and circles.

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

We here see that increasing or decreasing in the ratio of the square is a property of numbers so arranged, whatever these numbers may represent; whether, in short, they be regarded as abstract or concrete. Again, the lines at right angles to this diagonal show that if the sum of any two numbers is equal to twice the square root of any other number, the product of the former numbers is less than the square of the latter by the square of half the difference between the numbers of which it is the product; thus—

$$6+8=2\sqrt{49};\ \text{but } 6\times 8=7^2-1^2.$$
$$5+9=2\sqrt{49};\ \text{but } 5\times 9=7^2-2^2.$$
$$4+10=2\sqrt{49};\ \text{but } 4\times 10=7^2-3^2.$$

One may smile complacently at the ignorance of the market-woman who did not know this, and sold her eggs at a loss, thinking that x^2 is equal to $(x-1)\ (x+1)$; but the same equation in another form may perplex himself; thus $5^2=(1+2+3+4)2+5$, $7^2=(1+2+3+4+5+6)2+7$. As our knowledge of the nature of numbers, their arrangements, powers, and relations, is increased, so do mysteries vanish. Who, that has not enquired into the cause, would not be astonished when first told that with only four weights we may weigh from 1 to 40? (1, 3, 9, 27.) Observe our prime factor here again, as we may have occasion to draw upon it in this position for the solution of another problem.

CHAPTER IV.

KEPLER'S LAW, SOLAR INFLUENCE, ETC.

NEARLY three hundred years ago Kepler found out that the cubes of the distances of any two planets are in the ratio of the squares of their periodic revolutions. Having an ulterior object in view, the ratios of the first, second, and third, to all the rest are here given.

DISTANCE.	DISTANCE.	DAYS.	DAYS.	ERROR.
$a^3=580055453$:	$b^3=3784545357$	: : 88^2 :	225^2	0
	$c^3=10000000000$	”	365^2	0
	$d^3=3534952839$	”	687^2	0
	$e^3=$ … …		…	…
	$f^3=1408350593418$	”	4336^2	3 above
	$g^3=86793725776646$	”	10764^2	5 ”
	$h^3=70587054784500$	”	30698^2	11 ”
	$k^3=270999420608139$	”	60149^2	23 ”
$b^3=3784545357$:	$a^3=$ As above	: : 225^2 :	88^2	0
	$c^3=$ ”	”	365^2	0
	$d^3=$ ”	”	687^2	0
	$e^3=$ ”		…	…
	$f^3=$ ”	”	4340^2	7 above
	$g^3=$ ”	”	10775^2	16 ”
	$h^3=$ ”	”	30728^2	41 ”
	$k^3=$ ”	”	60208^2	82 ”
$c^3=10000000000$:	$a^3=$ ”	: : 365^2 :	88^2	0
	$b^3=$ ”	”	225^2	0
	$d^3=$ ”	”	687^2	0
	$e^3=$ ”		…	…
	$f^3=$ ”	”	4331^2	2 below
	$g^3=$ ”	”	10753^2	6 ”
	$h^3=$ ”	”	30665^2	22 ”
	$k^3=$ ”	”	60086^2	40 ”

Squares and cubes are associated with the ratios of both time and space; and are also the representatives of the surface and solidity of spheres. When one surface is compared with another surface, or one solidity with another solidity, the ratio, being the same for any spherical surface, and a different ratio being the same for any spherical solidity, may be left out. Thus, if a circle having a radius of 5, is compared with another circle having a radius of 7, the surface of the one is to that of the other as 5^2 to 7^2, the multiplication by 3·1416 not affecting the relation. In the same way, if a sphere having a radius of 5 is compared with another sphere having a radius of 7, the solidity of the one is to that of the other as 5^3 to 7^3, the multiplication by 2·0944 being common to both. On the other hand, if surface and solidity are compared, the ratio must be taken into account. Leaving this point as settled, we proceed to another.

If the distance of an object from a lamp, taking light for an illustration, be diminished to one-half, four times the former light is thrown upon it; and, if the distance be doubled, only one-fourth of the same. The question has been asked, taking the latter instance, "Where has the lost light gone?" Suppose the lamp suspended in the air in a dark, still night. The light emanates from it in a straight line to every object around it; that is, in other words, the distance between the lamp and the object is the radius of a great circle on a sphere. Thus, every point to which the light emanates is a point on the circumference of a circular plane, having the lamp for its centre. The lost light has gone all round the space enclosed between the former circumference and the present; hence, the ratio of the square.

Having laid down this theory as the foundation stone, a more benign theory of solar influence than any hitherto put forth may rest on it. The light wells forth from the sun as the centre, the undulations being propagated in widening concentric circles. Let the radius of the Inner Circle be 1, and that of the Outer 2. The same stream that crosses the inner circumference crosses the outer with the same velocity. A spongy ball rolling round each circumference with a uniform velocity, drinks up the same quantity from each in the course of a circular revolution. But the ball makes two revolutions of the Inner Circle for one of the Outer. Hence it drinks up twice as much from the former as from the latter. This difference is proportionally lessened if the velocity of the ball on the outer is less than on the inner. It is farther reduced if the ball on the outer is more porous than on the inner, and farther still if it is also larger. The application is obvious. Obliquity of angles, cones, discs, and spheres, have nothing to do with the question; otherwise the light does not proceed in the inverse proportion of the square of the distance, which we have found to be the areas of concentric circular planes. If size, density, atmosphere, and every other factor could be fairly calculated, we should not be surprised to find that the aggregate share of solar influence is the same to all, difference of organization compensating for difference of intensity. This theory has already the testimony of actual observation in its favour. For example, the snow on Mars is seen to melt as readily as with us. We may indeed cease our wailing for our benighted relatives.

It greatly strengthens the superincumbent theory of the Planetary Distances that the same investigation brings out a result so tangible as the ratio of the distance to the periodic revolution. It is the complement or counterpart of Kepler's Third Law. He found out the relation of the one planet to the other; whereas this ratio shows the relation to each other of the time and distance of the same planet. The process is seen in embryo in the prime factor 3·9; the 3 representing the cube of the distance and the ·9 the square of the days divided by 10; again, the 3 for the cube divided by 4, the first number representing the square. The ratio is thus $\frac{3}{4}$; but that is found by trial to be rather high. Referring again to our supposed Celestial Arithmetic, the nearest number corresponding to $\frac{3}{4}$ is ·749; that is, as with the prime factor, the number 7, with its square appended. But this number is found by trial to be about as much too low as the other is too high, the true results in even days being brought out by ·7496. This is on the basis of Bode's Law or Method II., beginning the distances with a unit (3) and its square (9). Putting these as one number, $x = 3$, and $x^2 = 9$; then $10x + x^2 = 39$; that is the number with its square appended. For example, by Method II. the Earth is 10. It must now be multiplied by 10 (100), and the cube of the product (1,000,000) divided by 10 (100,000), and this divided by ·7496, the quotient being the square of the days. The regulating ratio by tens may in practice be disregarded farther than to draw on them to the extent required.

To find the Days, the Distance being given.

Divide the cube of the distance by 7496, adjust the regulator, and take the square root of the quotient.

DISTANCE.		DAYS.	ERROR.
$a^3 =$ 580055453 ÷	7496	88^2	0
$b^3 =$ 3784545357 ÷	„	225^2	0
$c^3 =$ 10000000000 ÷	„	$365^2 +$	0
$d^3 =$ 35534952839 ÷	„	687^2	0
$e^3 =$ ……	…	…	…
$f^3 =$ 1408350593 ÷	„	4333^2	0
$g^3 =$ 86793925776 ÷	„	10759^2	0
$h^3 =$ 70587054780 ÷	„	30687^2	0
$k^3 =$ 27099942060 ÷	„	60126^2	0

If there is another planet beyond Neptune it is likely to run thus:—

$l^3 = 21680000000 \div 7496$ 170,000

To find the Distance, the Days being given.

Multiply the square of the days by 7496, adjust the regulator, and take the cube root of the product.

	Days.				Cube of Distance.
$a=$	$87{\cdot}969^2$	×	7496	=	5800847064.
$b=$	$224{\cdot}70087^2$	×	„	=	37847636545.
$c=$	$365{\cdot}25^2$	×	„	=	10002308850.
$d=$	$686{\cdot}9796^2$	×	„	=	35372695170.
$e=$	...				...
$f=$	$4332{\cdot}5848^2$	×	„	=	140815576784.
$g=$	$10759{\cdot}2198^2$	×	„	=	86774303700.
$h=$	$30686{\cdot}8212^2$	×	„	=	710056324814.
$k=$	60126^2	×	„	=	2710905862650.

If it were not for Jupiter and Saturn 7495 would be closer. This is probably owing to the great size of these planets. It is, no doubt, a significent fact that the ratio is the square of the orbit of what our theory makes the inmost planet, if there are intra-Mercurial planets, decimal points, as will be shown, having no place in Celestial Arithmetic. While this method of finding the days by the distance, and the distance by the days, may at first sight appear to be merely numerical calculation on the basis of Kepler's Third Law, the remarkable facts remain that Kepler's Law diverges from the truth in proportion to the distance while this does not, and the former requires no particular unit of measure or regulating ratio, while the latter does.

[The true mean distance of Mercury is 3·87099, but when set on the square the distance is 3·9, the difference being ·02901. Now, $\frac{{\cdot}02901}{3{\cdot}87099}$ is ·00749+, that is, the ratio of time and distance divided by 10^2. We do not yet see how to turn this to account farther, than that it is a fine example of coincidences which cannot reasonably be regarded as accidental. It is remarkable that this ratio ($\frac{3}{4}$) corresponds to the absence or omission of leap year—three times in four, while the rectification of setting the prime factor on the square corresponds to the same absence or omission in the even centuries.]

CHAPTER V.

DISCREPANCIES—CELESTIAL ARITHMETIC.

FINDING that the true mean distance is not brought out exactly by either square or circle, we may now inquire into the cause of the discrepancies.

a = 3·9	·029 above mean distance.
b = 7·2	·033 below „
c = 10·025	·025 above „
d = 15·21	·027 below „

It thus appears that the planets, when thrown off from the surface of the inflated rotating molten sphere, were not on the mean, but on one or other extremity of the swing. Here b = 7·2 is ·03332 below the mean distance. Taking 3·9 as radius of fig. 1, S r is equal to 2·758 and $r\,v$ to ·5711, making b = (3·9 + 3·329) = 7·2329. How wonderfully near this comes to the true mean distance, 7·23332. Again, adding to a (3·9) the arc of its quadrant 6·125, makes C = 10·025. In this way C − S r (10·025 − 2·758) makes b = 7·267; but the average between 7·2 (as by fig. 2) and 7·267 (as by fig. 1) is 7·2335, which is also very near the true mean distance.

Now, as C is here found by adding to a, the arc of its quadrant, so d may be found by adding to a the arc of quadrant b, that is, the arc of quadrant of which b (mean distance) is radius: thus, 3·9 + 11·3621 = d = 15·621; but, by the square, d = 15·21. The average or mean distance between 15·621 and 15·21 is 15·2361, the true mean distance being 15·23691. This approximation is astonishing.

Here, unfortunately, we lose our guide, the true mean distance, as it cannot be known whether all the Asteroids are discovered or not. The second proportional evidently gives the distance too great, as the third proportional makes Jupiter's distance too little.

Jupiter's distance, as given by Methods I. and II., is below the mean, and the square of Venus (true mean distance) is above it. The average of these (51·84 and 52·32) is 52·08, the true mean distance being 52·02798.

Taking the sum of *d*, *e*, and *f* (fig. 2) for one extremity, 95·13, and the cube of Venus divided by 3·9 for the other, the mean distance is 95·4 for Saturn. We have already discussed the eighth distance; but several other ways of accounting for it will afterwards be shown.

Returning to the Asteroids, we find a great number of approximations about 27·6; thus, $\mathrm{S}r \times 10 = 27{\cdot}58$; $(c - b) \times 10 = 27{\cdot}668$; $d + \sqrt{c^2 + b^2} = 27{\cdot}579$; the circumference of the Inner Circle taken from Jupiter $= (52{\cdot}02798 - 24{\cdot}5) = 27{\cdot}52798$; $(d - c)^2 = 5{\cdot}237^2 = 27{\cdot}426$; by Method I. $= 27{\cdot}437$.

There are also various approximations about 28. By Method II. 28·08 and 27·92; square root of diameter multiplied by 10 = 27·92; and by Celestial Arithmetic from 27 to 27·9. Such a convergence of distances seems to point to the completion of a cycle or series of revolutions, or what may be called a swivel on the chain; and the enlarged powers of the next planet (Jupiter) may thus be accounted for. Again anticipating, it may here be stated that this opinion will be afterwards more strongly confirmed.

Let us now look a little more into our Celestial Arithmetic. Supposing the solar system to be a great sphere, it may be supposed to be subdivided into concentric spheres. Again, for convenience, we may substitute the circle for the sphere, as in fig. 1, the square of the immediate Outer Circle being twice that of the Inner, that is, in other words, the square of the circumscribed circle being twice that of the inscribed. It has already been observed that Bode's Law is partly founded on this geometrical fact, the side and diagonal of the square bearing to each other the same relation.

Larger Table of Celestial Arithmetic.

The first column begins with what will afterwards be shown to be the diagonal of a geometrical square unit second column gives the arc of quadrant, for which the first is radius; and the third gives the square of the first.

	I. Radius.	II. Arc of Quadrant.	III. Square of Radius.
1.	1·379	2·166	1·9 = $39^2 \div 2^3$
2.	1·95	3·0625	3.8 = „ $\div 2^2$
3.	2·758	4·332	7·6 = „ $\div 2$
4.	3·9	6·125	15·21 = „
5.	5·516	8·66	30·42 = „ $\times 2$
6.	7·8	12·25	60·84 = „ $\times 2^2$
7.	11·03	17·32	121·68 = „ $\times 2^3$
8.	15·6	24·5	243·36 = „ $\times 2^4$
9.	22·06	34·65	486·72 = „ $\times 2^5$
10.	31·2	49·	973·44 = „ $\times 2^6$
11.	44·12	69·3	1946·88 = „ $\times 2^7$
12.	62·4	98·	3893·76 = „ $\times 2^8$
13.	88·24	138·616	7787·52 = „ $\times 2^9$
14.	124·8	196·	15575· = „ $\times 2^{10}$
15.	176·48	277·232	31150· = „ $\times 2^{11}$
16.	249·6	392·	62300· = „ $\times 2^{12}$

As the Planetary System is here, by our theory, completed, the table need not be farther extended. Column III. circulates at the 11th line, again at the 21st, 31st, and so on, each circulate being increased 1024 times, that is, by the 10th power of 2, or 32^2. The number 32 figures prominently in physical science, literally meeting us in the teeth—32 in number. A writer on natural history has recently shown that even a tuft of moss is arranged on this number. So are the thermometer and mariner's compass. By the system of arithmetic now being sketched, the 3 represents the three sides of the cube, which multiplied together give its solidity; and the 2 represents the two sides of the square, which multiplied together give its area. The number 32 is the ratio of the square, as 3·1416 is that of the circle. The regulating ratio is the mean; thus—$\sqrt{32 \times 31416} = 1000 +$, the + going to the next circulate. How beautifully this illustrates the properties of the circle and square in bringing out the natural number between them!

Column II. circulates at the 21st, 41st, 61st line, etc., each circulate being increased 1024 times. Column I. circulates and increases in the same way. On comparing Columns I. and III., the first line is seen to be the beginning of the table; thus, the corresponding number, increased by $\frac{1}{39}$ is one line down in Column I., the numbers being 1·9 in Column III., and 1·95 in Column I. The next pair of correspondents are two lines apart, the next three, and so on. On comparing Columns I. and II., the correspondents are uniformly found 12 lines apart, the latter increased by $32 \times 3{\cdot}1416$.

Selecting only what bears on our present enquiry, we may now look across the lines. To mark revolution, the radius and arc of quadrant should circulate; and so they do. When the arc becomes the radius, the radius, by means of the regulating ratio, becomes the circumference of the circle. For example, 3·9 in the foregoing table is the radius, and 6·125 is the arc of quadrant. By making 6·125 the radius, the circumference is 38·4846. Going down the columns is like going down the stream with wind and tide, the ratios of circle and square being on the same side ($32 \times 3{\cdot}1416$), but in crossing the line they are on opposite sides $\left(\frac{32}{3{\cdot}1416}\right)$; hence $38{\cdot}4846 \times \frac{32}{3{\cdot}1416} = 392$, as found in the next circulate, 12 lines down. To make this clear, let us take another example. If the radius is 100, the arc of quadrant is 157·08. The latter number is in turn the radius of a circle 986·96 in circumference. The average of this number and the square of 32 is $\left(\frac{986{\cdot}96 + 1024}{2}\right) =$ 1005·48, while the other way ($32 \times 31{\cdot}416$) is 1005·312.

The liberty just taken with the decimal point in the ratio of the circle prepares the way for the discovery that the progression 3, 9, 27, is the ratio of the circle divided by 8 $\left(\frac{31416}{8}\right)$, as shown in fig. 1. Decimal points have no place in Celestial Arithmetic, the square and circle providing a regulator.

The first circulate of the radius 3·9 ends with the 15th line. Here, making the arc 277232 become the radius, the radius 176492 becomes the circumference. To save the trouble of reducing it, we have only to look for its correspondent, 8 lines up in Column II., where we find 17327. This, it will be seen, is the circumference of the inscribed circle, fig. 1, whose radius is S*r*. The square of this number is the distance of Neptune, 300·225. The 16th line, dealt with in the same way, gives the position of the ultra-Neptunian planet, if there is one. Reverting to 277232, we see that the mean distance of the Asteroids in Column I., 12 lines intervening, is 2758, corresponds to the radius of the inscribed circle (17327), and comes out exactly.

CHAPTER VI.

INTRA-MERCURIAL PLANETS.

AMONG the many wonderful properties of our theory, its elasticity is not the least remarkable. It may, indeed, be *Vulcanised* without sustaining the slightest injury. Looking again at the Shorter Table of Celestial Arithmetic (13927), on which we have already drawn so freely, we see, that while the number 39 begins the ratio of the circle divided by 8, the number 1309 does the same when the ratio is divided by 24, the cipher merely marking revolution.

As an instance of the manner in which these symbols combine and circulate, multiply the first two by 6 (13 × 6), and the third separately (9 × 6), a cipher or revolution intervening (1309 × 6), and the same division of the ratio of the circle is obtained as in multiplying all the symbols except the first by 2 (3927 × 2) = 7854. Again, multiplying the first three by 2 (139 × 2) = 278, we get the beginning of the next number in the progression (81). And not only do the symbols combine and circulate, but, in circulating, they also *grow*, like the rotating snowball, adding a fresh layer at every round. Thus, 139 rises in the next circulate to 142, and 270 to 276.

Even the formation of the Arabic numerals, whatever is their origin, is highly symbolical. It may be true that the rose by any other name would smell as sweet, but these numerals in any other form would not be so expressive of their nature and properties. The form, in some of them at least, may be regarded as essential, and not accidental. Thus, rotation is strikingly indicated by the cipher (0); the square or right angle by the first square number (4); and the double of that number, or of the ratio of the circulate of the square, is represented by two circles conjoined, like a swivel; thus, $8 = 4 \times 2$; $8 = \sqrt{32 \times 2}$.

New light has thus been thrown on the relationship of circle, square, and number. We can form no conception of such harmony with any other notation than decimal. It would be a great boon to our country, though no doubt causing temporary inconvenience, if our tables of money, weights, and measures were changed to the decimal system. On the other hand, we have only to look at the foregoing columns to observe that calculating by the dozen and the score is also founded on nature, the circulates at 13, 25, 37, etc., and at 21, 41, 61, etc., being evidence. We need not stop to show how largely these numbers, their divisions, and multiples figure in our present tables. Indeed, the more we enquire into the existing order of things, in this or in any other direction, the more we see natural law prevailing. But, to return from this digression, the number 139 may be the distance from the Sun of an intra-Mercurial planet. Reverting to the Larger Table, we find the corresponding number to be 1379, that is, one-half of the number we have been discussing (2758), or, the distance S *r*, fig. 1, bisected. Taking 1385 as the average, and disregarding the decimal point, as already explained, the distance of Uranus is the square (1918). It is said that several witnesses, at sundry times, have shown that the distance of Vulcan—the supposed planet—is about 13 millions of English miles from the Sun. Now 1385, or 1379, multiplied by 95, would be a little above that distance; but as the statement adds that the period of revolution is about 20 days, the distance of the Earth, as the unit of measure, appears to be taken at about 92 millions, for 19 days are above the period at 95. The reason for taking the Earth at 95 instead of 92, or any other number, is to keep on the square, just as we have to take Mercury at 39 instead of 3871; for, as shown by the Larger Table, 95 is equal to $39^2 \div 2^4$. We have three ratios to deal with, namely—

32 = Ratio of circulate of geometrical square.
31416 = do. of circle.
$\sqrt{10} = 3162$ = do. of circulate or notation of number.

Hence arises the necessity of keeping on the square. When thus yoked, the numbers work harmoniously; but otherwise they prove recalcitrant. Here is a fine example, showing one vein of relationship running through the whole—

$$\text{As } k=h+\sqrt{g^2+f^2}\text{; so } e=d+\sqrt{c^2+b^2}$$

Let us now begin with a geometrical square of 1.

The diagonal is $\sqrt{2} = 1414$.

Proceeding now to a square having 2 for its side. The diagonal is $\sqrt{2^3} = 2828$. Reducing these distances, as shown by the Larger Table, the first is 1379, and the second 2758. The latter number is the distance S r, fig. 1, as already shown, and it bears the same relation to Neptune that Mercury bears to the ultra-Neptunian planet; thus, $27{\cdot}58^2 : 39^2 :: 300 : 600$.

It is also seen that the orbit of the inmost planet (866) is the square root of the ratio of time and space ($\sqrt{7496}$). Taking now this distance as the side of the square, that is, doubling the square, the diagonal is 3·9, the third distance.

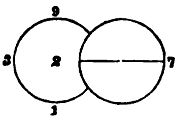
As Vulcan's distance is 139 in the Summary or Shorter Table, so, reading straight on, the number 27 represents the distance of the planet nearer Mercury, the more exact distance being shown in the Larger Table. If there is one planet nearer the sun than Mercury, there are certainly two. It is indeed likely that the transit of each has often been seen, and that observers have been put off the scent of both by the one being mistaken for the other, two not being dreamt of.

The mean distance of the inmost appears to lie between 1379 and 1442, the latter number being Uranus divided by the third proportional (13·3); and that of the other between 271 and 279. We are still unable to account particularly for all the relations and combinations symbolising the planetary distances; but the following order, when carefully examined, will be found to show a regular sequence, it being kept in mind that the cipher as marking revolution, may be drawn on to the extent required in every distance.

The diagram represents the swivel. Go round on the circumference and across on the diameter in the order of the symbols in the table. In the second round, as indicating revolution on the swivel, go from 9 to 7, and in the third from 9 to 2 as at first. Then turn and return once and again.

1. 139
2. 271
3. 39
4. 72
5. 1
6. 39^2
7. 27 (1 × 3 × 9) 1st cycle.
8. 729^2
9. 31^2
10. 139^2
11. (Circumf. of Rad. $271)^2$.
12. (Circumf. of Rad. $39)^2$.
13. Continual product, 1 × 3 × 9 × 2 × 7 (complete cycle).

(13927)



The eleventh distance, taken the one way, is 271 for radius, and taken the other 279; thus giving the mean 275. This distance is given more directly by another regular sequence; thus, the first and last, the one next each (17329^2). Here, again, the last distance—the orbit of the ultra-Neptunian planet—indicates in a remarkable manner the completion of the system. The distinctive position of the Asteroids is here finely shown.

[While the *time* of the end of the present system is known to no man, the *manner* is plainly foretold. The reference to Scripture with which Chap. II. concludes is therefore justifiable and appropriate Searching the Scriptures for the number of the planets, we find in Joseph's dream "*the eleven stars*" bearing a family relation to each other; and from both the interpretation and the preceding dream, it seems to be implied that Joseph himself was represented by the *twelfth*. We find also in John's vision "a woman clothed with the sun, and having on her head a crown of *twelve stars*." If the expression has, as we may reasonably suppose, a literal signification, what can it be but the planets? The Asteroids are, as it were, the scattered fragments of one planet, and their average distance from the Sun marks one planet's place.

To attempt, however, to prove our theory from such obscure figurative expressions would be weak and presumptuous.]

CHAPTER VII.

FORCE AND MOTION.

FORCE, as our supposed physical cause, first sets the wheel in motion; and then the motion produced works along with force, in other words, is changed into force, and works along with it, in producing greater force and greater motion. Suppose an invading army first overcoming a detachment of the enemy, and increasing its own strength by getting the conquered to swell its ranks and fight against their own country; and so on with one detachment after another, till the whole country is subdued. This is an illustration of the working of force and motion as shown in Chapter II.

A stone cast into still water is an illustration of the work produced by force and motion, and shown by concentric widening circles. The way of the wind—a recent discovery—is a somewhat similar illustration, advancing by widening circles, and not in a straight line, as was formerly supposed.

The work not transferred to the Middle Wheel may be illustrated by a ball struck by a force sufficient to make it move round so far on the pole of its axis but not to advance.

Suppose a learner beginning Euclid. By mental effort, that is, force of mind, he acquires the knowledge of the first proposition. This knowledge strengthens him for overcoming the second, which again increases his power for the third. Similar illustrations may be given in any branch of learning. This is the same law of nature prevailing alike in mind and matter. When we speak of the *powers* of numbers we only express a literal fact. The force of numbers is increased or diminished by their position and relation. The changes in their position constitute their motion. Take, for example, the number 47. The 4 has here the force of 40. Where is the motion? The cipher is the pivot, pole, or swivel, on which the 4 revolves, and is only concealed by the 7 being on it. As it is with the planets, so it is with the digits, the pivot being the Sun —the cipher—respectively.

The revolution of the Inner Circle, fig. 2, corresponds to the multiplication of the unit 5 by 2; for in transferring the *distance* made by it to the Middle Circle, we leave 0 and carry 1. In transferring from the Middle to the Outer Circle, the multiplication goes by the square, 5×2^2, leaving 0 and carrying 2.

It was observed in laying our foundation stone, that the nearer the lamp the stronger the light, and that the lamp is the centre from which the light emanates. So it is with numbers. The force that any two numbers exert on each other is in the inverse proportion of the square of their distance from the centre. Take, for example, 7 and 17. The mean or central number between them is $\left(\frac{7+17}{2}\right) = 12$, and the difference of each from this centre is 5; hence $12^2 = (7 \times 17) + 5^2$. The force of the numbers is 119 $(144 - 119) = 25$ being lost by distance from the centre. Bring them 1 nearer the centre; then $12^2 = (8 \times 16) = 128 + 4^2$. Their force is now 128, only 16 being lost by distance from the centre. Again, 1 nearer; then $9 \times 15 = 135$, the loss by distance being only 9; the next approach, $10 \times 14 = 140$, with the loss of 2^2; and so with any two numbers. It will be seen that the top-stone now reared is nothing else than the well-known theorem $x^2 = (x - y)(x + y) + y^2$. Thus applied, geometrically or numerically, it may be called the "Force of Factors." Here, again, we see that the law of nature is the same with numbers, as with mind and matter. When applied to the measurement of surface, the factors, like those in the problem of the Planetary Distances, require to be set on the square, that is, at right angles. The square of the second proportional is equal to the product of the first and third, $7{\cdot}2^2 = 3{\cdot}9 \times 13{\cdot}3$. Now the mean of these factors $\left(\frac{3{\cdot}9 + 13{\cdot}3}{2}\right)$ is 8·6, the square of which is the central force, the distance of the factors being 4·7. Hence $8{\cdot}6^2 = (3{\cdot}9 \times 13{\cdot}3) + 4{\cdot}7^2$. or, $7{\cdot}2^2 = 8{\cdot}6^2 - 4{\cdot}7^2$.

It may now be shown that by taking 4 as the side of the fourth square, an approximation may be got in this way for the fourth distance. Thus, $\sqrt{4 \times 4 \times 2} = \sqrt{32} = 5{\cdot}66$. Then $5{\cdot}66 - 3{\cdot}9 = 1{\cdot}76$ (distance from centre). Now, $1{\cdot}76^2 = 3{\cdot}1$ (loss of central force). Hence $32{\cdot} - 3{\cdot}1 = 28{\cdot}9$. This number reduced as before $(28{\cdot}9 \times \frac{1000}{1024} - 28{\cdot}2)$, of which number 3·9 being the one factor, the other $\left(\frac{28{\cdot}2}{3{\cdot}9}\right)$ is 72·3.

We now see that it is not a mere fancy, but a plain literal fact, that the Summary or Shorter Table of Celestial Arithmetic rotates or circulates on the square as well as the Larger Table; thus—

1 3 9 2 7
$27^2 = 729$
$139^2 = 193$

What is called the law of gravitation has hitherto been regarded as confined to matter, being extended to the mind in only a figurative sense, and not applied to numbers at all. Exception should perhaps be made of the latest methods invented for special calculations—Fluxions and

Quaternions—in which the force or motion of numbers is assumed merely as a convenient hypothesis. Both Newton and Hamilton viewed the science of number through the mist. .They, indeed, saw, as it were, the becalmed vessel; but there appeared to be neither breath nor motion; and so they regarded it merely as—

> "A painted ship upon a painted ocean."

The latter of these methods goes farther than the former in the direction of leading up from appearance to reality—from similarity to identity. This, again, is just the order of nature; for much of our knowledge from observation comes to us in this way. An object is seen in the distance. It gradually approaches. It is a human being,—a man; nearer, he looks like our friend; his walk, size, dress, and general appearance; nearer still, his features—Hail! it is he.

It must be admitted, however, that in propounding the reality of the force and motion of lines and abstract numbers, we are treading as closely on the outmost boundaries of transcendentalism as one can safely venture.

But whatever this property of nature may be named, it prevails alike in material bodies, geometry, and number; in the solar system, the rectangular field, and the multiplication table.

There is no doubt an intimate connection between the size and density of the planets, and their distances and periods of revolution. The density, for example, is seen to bear a certain modified inverse proportion to the distance. Our mine is far from being exhausted; and the prosecution of the investigation in this direction is likely to be rewarded by the advancement of physical science. This part of the subject, however, cannot be entered on at present.

"*Who hath measured the waters in the hollow of His hand, and meted out heaven with the span, and comprehended the dust of the earth in a measure, and weighed the mountains in scales, and the hills in a balance.*"

NOTE FOR THE DIAGRAMS.

To prevent the diagrams conveying a wrong impression to the mind of the reader, it should be clearly understood that the scale is not uniform. The Outer Circle on the same scale as the Inner would cover the top of an ordinary round table.

"To give a general impression," says Herschel, "of the relative magnitudes and distances of the parts of our system, we may choose any well-levelled field or bowling-green. On it place a globe two feet in diameter: this will represent the Sun; Mercury will be represented by a grain of mustard seed, on the circumference of a circle 164 feet in diameter for its orbit; Venus, a pea, on a circle 284 feet in diameter; the Earth, also a pea, on a circle of 430 feet; Mars, a rather large pin's head, on a circle of 654 feet; Juno, Ceres, Vesta, and Pallas, grains of sand in orbits of from 1000 to 1200 feet; Jupiter, a moderate-sized orange, in a circle nearly half a mile across; Saturn, a small orange, on a circle of four-fifths of a mile across; and Uranus, a full-sized cherry or small plum, upon the circumference of a circle more than a mile and a half in diameter. As to getting correct notions on this subject by drawing circles on paper, or, still worse, from those very childish toys called orreries, it is out of the question."

POSTSCRIPT, 1884.

CORRECTION OF THE TESTING OF KEPLER'S THIRD LAW (page 15).

The even days have been taken, 88 instead of 87·969, 225 instead of 224·7, and 365 instead of 365·25. This error slipped in unobserved, from the first of these periods being so near the even days; and afterwards only the results, without reflecting on how they were found, were, by an unaccountable oversight, compared with the complement (page 19), calculated correctly. There is no great cause for regret, as this correction is likely soon to reach the few copies of the book that are in circulation, and the farther investigation to which it has led throws a flood of light on the whole subject, like the removal of a cloud obscuring the sun. Palpable as the error is, it was not detected till an attempt was made to supersede the said law by finding one that would not diverge in proportion to the distance; and after much careful thought the following conclusion was come to:—The mean distance of one planet, regarded as the radius of its orbit or circumference, is to the mean distance of another planet as the circumference of the one is to that of the other. But the radius also measures time, and is therefore a pendulum; hence the square root of the distance of the one is to the square root of the distance of the other as the period of revolution of the one is to that of the other. The question thus resolves itself into one of compound proportion.

Example.

Venus.		Uranus.		Days.
·723332	:	19·182639	: :	224·7
$\sqrt{\cdot 723332}$		$\sqrt{19\cdot 182639}$		

The most convenient method of solution is the following:—$\frac{19{\cdot}182639}{{\cdot}723332} = 26{\cdot}519826$, the square root of which is 5·1497; then $26{\cdot}519826 \times 5{\cdot}1497 \times 224{\cdot}7 = 30687{\cdot}087$, the true number being 30686·8212. The square root thus found (5·1497) is the cube root of the quotient of the days; hence the process is reversed thus, $\frac{30686{\cdot}8212}{224{\cdot}7} = 136{\cdot}56796$, the cube root of which is 5·1497; then $5{\cdot}1497^2 \times {\cdot}723332 = 19{\cdot}182639$.

On finding that this is only Kepler's Third Law in another form, the error was detected; and this way of splitting it into factors led to the conclusion that the ratio of the complement (·75) is the square of the radius of a cubic unit, the truth of which is here proved in the sequel.

THE RELATION OF DECIMAL NOTATION TO THE GEOMETRICAL SQUARE.

Let $a, b, c, d,$ represent successively the four sides of a square. Then

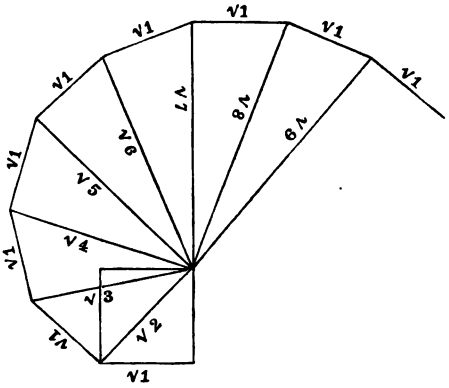
$$\begin{aligned} a&=1\\ a+b&=2\\ a+b+c&=3\\ a+b+c+d&=4\\ \hline \text{Sum}&=10 \end{aligned}$$

The four measures thus found (1, 2, 3, 4) form decimal notation; thus $4+1=5$; $4+2=6$; $4+3=7$; $4+3+1=8$; $4+3+2=9$; $4+3+2+1=10$. Instead of measures thus applied as might be necessary in certain cases, if we suppose the four numbers to be weights, the illustration will be more familiar; for with these four weights we can weigh from 1 up to 10, and no more, without repeating the process.

These Arabic symbols, even in their present form, show well-defined traces of their origin; thus—

One side of square,	-		=1
Two sides do.	-		=2
Three sides do.	-		=3
Two sides and diagonal,			=4

Again—



Begin with the side 1 and make a square on it. Draw the diagonal $\sqrt{2}$, and at one end of it draw out the side of the square for perpendicular. Form the hypothenuse by lifting the base and side as laid down, placing the base for hypothenuse, and making up towards the centre what is wanted from the remainder of the square. The hypothenuse $\sqrt{3}$ now becomes in turn the base of the next triangle, which exhausts three sides of the square, its base being $\sqrt{4}$ and its perpendicular $\sqrt{1}$. Proceed in the same way till the fourth side of the square, from which the upmaking to the hypothenuse is successively taken, is exhausted, and no more can be added to the base to make it the hypothenuse. This takes place on reaching $\sqrt{9}$ for base and $\sqrt{1}$ for perpendicular. The hypothenuse $\sqrt{10}$ is the side of a square of 10, and begins a new circulate, which must be dealt with in the same way as the former, being regarded as *one* ten, or a unit, ten times the size of what we began with.

Looking at multiplication table as given on page 14, we see that the factors are, leaving out the symbols, first made up of square units arranged in rectangles, and then resolved into lineal measure by the square units being drawn out into a chain, the length of which only is taken into account. In the same way the surface of a sphere is 3·1416 square units when the diameter is the side of each, and these drawn out are the measure of the circumference. In concentric widening circles, as seen when a stone falls into still water, the half of the wave is on one side of the diameter, and the other half on the other side. The diameter is 1, and the measure of the surface of the semicircle is ·3927 of a square unit, which becomes a measure of length on the circumference. Now, a sphere is generated by the revolution of a semicircle on its diameter, the process being thus symbolised by 13927. In this way the wonderful properties of that number, as shown on page 27, may be accounted for.

Let us now look at the diagram as representing cubes instead of squares, and suppose each cube to be enclosed in a sphere. The radius of the first is 86602 +, the square being $\frac{3}{4}$, or ·75. The square of the radius of the second is ·75 × 2, the third ·75 × 3, and so on up to the tenth, the square of the radius of which is 7·5. This is a decisive proof that any other notation than decimal would not be in harmony with nature, and would neither give the ratio of the cube nor that of the distance of any planet to the period of its revolution. Beginning with units as shown under Method II., this law, which entirely agrees with and is the true complement of Kepler's Third Law, may be thus stated:—

> As the square of the radius of a cubic unit (·75)
> Is to the square of its numerical period (10^2),
> So is the cube of the distance of any planet
> To the square of the period of its revolution.

Time, space, and number, revolve in harmony by squares and tens, and ·749 (that is, ·7 with its square divided by 10) and ·75 may be regarded as the boundaries on each side of the highway. The radius is drawn from the centre to one of the corners. Suppose another radius drawn to the remote corner, on the same side or face of the cube. A whirlwind arises in the centre of the cube, and its centre runs along on one radius, while the part of its circumference within the cube is bounded by the other radius. On reaching the surface, the measure of the circumference is to the radius of the cube as the area of the decagon is to $\frac{3}{4}$ of the square—

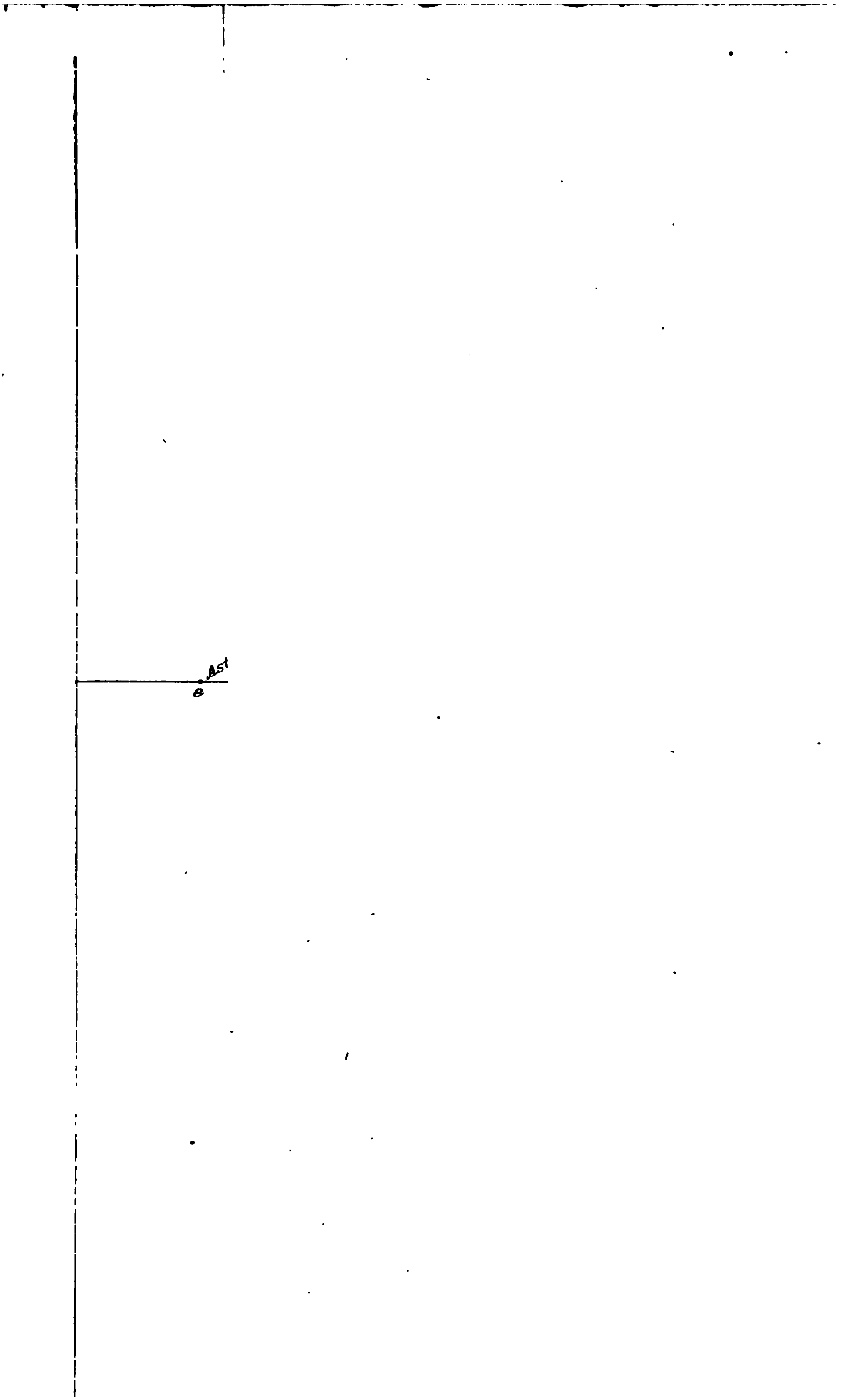
$$8{\cdot}8857 : 7{\cdot}6942 :: {\cdot}866 : {\cdot}75.$$

This is the same relation of tens and squares in another form.

We may here see the natural way of doubling the cube. Lift the diagonal of the cube, and lay it on the diagonal of its face. The former extends (1·7320508 – ·7071068) 1·024944 beyond the radius or centre. Lay this extension, with the radius of the cube added (1·024944 + ·866025) on the side of the cube. The line extends ·890968 beyond the side. Then $\cdot890968^2 \times 2 = 1{\cdot}26^2$. This is not strictly numerically accurate, the true measure being ($\sqrt[3]{2}$) 1·259921; but the gain does not amount to $\frac{1}{18}$ of a farthing in the pound, so to speak. This is sufficiently accurate for all practical purposes, and would no doubt have satisfied the Delian Oracle. So also, $1+\left(\frac{1\times2}{10}\right)+\left(\frac{1\times2\times3}{10^2}\right)=1{\cdot}26.$

If the side of a square is made the side of a decagon, the remaining three sides, each increased by ·0259, form the diameter of the inscribed circle. Divide this increase by 10; then $1{\cdot}00259 \times 2$ is the radius of a circle, of which the tenth part is the measure of the side of the double cube (1·2599). The construction is obvious.

As it is not yet known when or whether a second edition of this work may be called for, the author would now have considered any criticisms; but, strange to say, only one, scarcely deserving the name, has come under his notice. A reply was sent to the newspaper in which it appeared, but the editor considered the discussion unsuitable for his columns. The reviewer, to save himself all unnecessary trouble, demolished the whole at one blow by showing, in his own way, that it is on a wrong foundation, the illustrations being by spheres and circles, with the sun in the centre, which Kepler's First Law shows to be nonsense. Our critic disproves too much; for it is here clearly shown, without going farther, that by the same argument Kepler's First Law would disprove his Third. It is to be hoped that what has been here advanced will, in the interest of science, give rise to discussion.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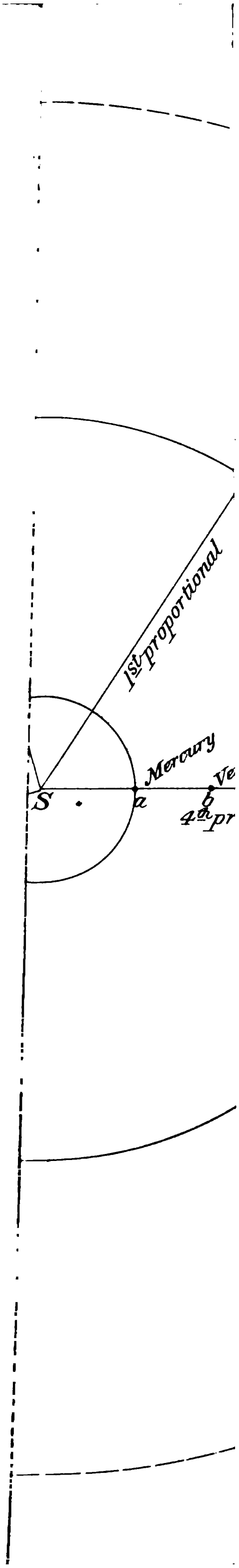
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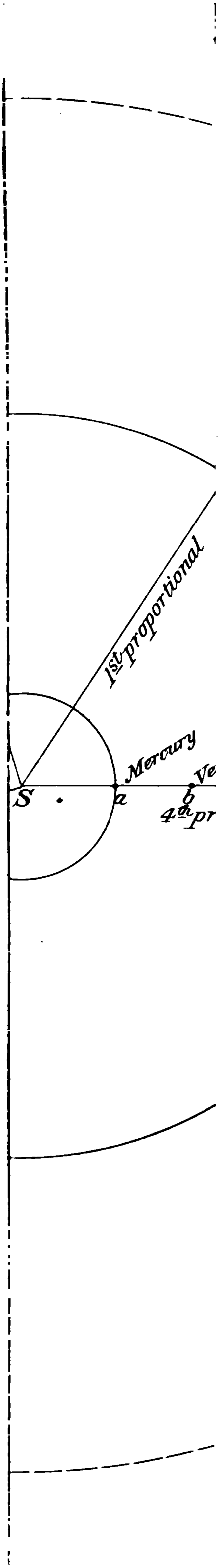
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